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MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **October/November 2025**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

DC (DE/SW) 343520/2

1 Find the set of values of the constant k for which the quadratic equation

$$3kx^2 + (k+8)x + 3 = 0$$

has two distinct real roots. [4]

quadratic eqⁿ has two roots if

$b^2 - 4ac > 0$

$(K+8)^2 - 4 \times 3K \times 3 > 0$

$K^2 + 64 + 16K - 36K > 0$

$K^2 + 64 - 20K > 0$

$K^2 - 20K + 64 = 0$

$K = \dfrac{20 \pm \sqrt{400 - 4 \times 64}}{2}$

$K = \dfrac{20 \pm \sqrt{400 - 256}}{2}$

$K = \dfrac{20 \pm \sqrt{144}}{2}$

$K = \dfrac{20 \pm 12}{2}$

$+ \quad - \quad +$

$4 \quad 18$

$K = 16$ $\qquad$ $K < 4$

$K = 4$ $\qquad$ $K > 16$

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2 A geometric progression has first term a and common ratio $\cos\theta$, where $0 < \theta < \frac{1}{2}\pi$. It is given that the second term is 8 and the fifth term is $\frac{1}{8}$.

(a) Find the value of θ. Give your answer correct to 3 significant figures. [3]

$a, \; r = \cos\theta$

$a_2 = ar = 8$

$= a \cdot \cos\theta = 8 \quad - 1$

$a = \frac{8}{\cos\theta} \quad - ①$

$a_5 = ar^4$

$= a(\cos\theta)^4 = \frac{1}{8}$

$a = \frac{1}{8\cos^4\theta} \quad - ②$

equate eq^n ① and ②

$\frac{8}{\cos\theta} = \frac{1}{8\cos^4\theta} = \cos^3\theta = \frac{1}{64}$

$\cos\theta = \frac{1}{4}$

$\cos\theta = \frac{1}{4}$

$\theta = \cos^{-1}\left(\frac{1}{4}\right)$

$\theta = 75.522$

(b) Find the exact value of the sum to infinity. [2]

$S_\infty = \frac{a}{1-r}$

$a = \frac{8}{\frac{1}{4}} = 32$

$S_\infty = \frac{32}{1-\frac{1}{4}} = \frac{32 \times 4}{3} = \frac{128}{3}$

$S_\infty = \frac{128}{3}$

$\frac{14}{2\times2}$ $\frac{4\times3\times2}{\cancel{4}\times}$

3 In the expansion of

$$(px+3)^5 - \left(x^3 + \frac{p}{x}\right)^4,$$

the coefficient of x^4 is 216.

Find the value of the positive constant p. [5]

$\left[{}^5C_0(px)^5 + {}^5C_1(px)^4 . 3 \ldots\right] - \left[{}^4C_0(x^3)^4 + {}^4C_1(x^3)^3 \frac{p}{x} + {}^4C_2(x^3)^2\left(\frac{p}{x}\right)^2 + {}^4C_3(x^3)\left(\frac{p}{x}\right)^3 + \ldots\right]$

$\left[px^5 + 5p^4 . x^4 \times 3 \ldots\right] - \left[x^{12} + 4x^8 . p + 6x^4 p^2 + 4p + \ldots\right]$

Coefficient of x^4

$5p^4 . 3 - 6p^2 = 216$

$15p^4 - 6p^2 - 216 = 0$

$5p^4 - 2p^2 - 72 = 0$

Solve for p $\quad p^2 = x$

$5x^2 - 2x - 72 = 0$

$x = 4 \Rightarrow p^2 = 4 \Rightarrow p = \pm 2$

$x = -3.6 \Rightarrow p^2 = -3.6$ Imaginary value dis card

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4 **(a)** Express $1-6x-x^2$ in the form $a-(x+b)^2$, where a and b are constants. [3]

$1-(x^2+6x)$

$1-(x^2+6x+9-9)$

$1-(x+3)^2+9$

$10-(x+3)^2$

(b) The graph of $y=x^2$ is transformed to the graph of $y=1-6x-x^2$ by a reflection followed by a translation of $\begin{pmatrix} m \\ n \end{pmatrix}$. Give details of the reflection and determine the values of m and n. [3]

$y=x^2$ is Transformed to

$y=10-(x+3)^2$

$y=-f(x)$

reflection in about x axis

and Translated by $\begin{pmatrix} -3 \\ 10 \end{pmatrix}$

5 **(a)** Show that $\tan^4\theta - 1 \equiv \dfrac{1-2\cos^2\theta}{\cos^4\theta}$. [3]

$$\frac{\sin^4\theta}{\cos^4\theta} - 1 = \frac{\sin^4\theta - \cos^4\theta}{\cos^4\theta}$$

$$= \frac{(\sin^2\theta - \cos^2\theta)(\sin^2\theta + \cos^2\theta)}{\cos^4\theta}$$

$$= \frac{\sin^2\theta - \cos^2\theta}{\cos^4\theta}$$

$$= \frac{1 - \cos^2\theta - \cos^2\theta}{\cos^4\theta} = \frac{1-2\cos^2\theta}{\cos^4\theta}$$

(b) Hence solve the equation $\cos^2\theta(\tan^4\theta - 1) = 7$ for $0° < \theta < 180°$. [4]

$$\cos^2\theta\left(\frac{1-2\cos^2\theta}{\cos^4\theta}\right) = 7$$

$$\frac{1-2\cos^2\theta}{\cos^2\theta} = 7$$

$$\frac{1}{\cos^2\theta} - 2 = 7$$

$$1 - 2\cos^2\theta = 7\cos^2\theta$$

$$1 = 9\cos^2\theta$$

$$\cos^2\theta = \frac{1}{9} \Rightarrow \cos\theta = \pm\frac{1}{3}$$

$$\theta = \cos^{-1}\left(\frac{1}{3}\right)$$

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6 Functions f and g are defined by

$$f(x) = (x+3)^2 - 12 \quad \text{for } x \geqslant 0,$$

$$g(x) = 2x - 5 \quad \text{for } x \in \mathbb{R}.$$

(a) State the range of f. [1]

The vertex is $(-3, 12)$ but domain is $x \geq 0$

so put $x = 0$ in $f(x)$

$f(x) = (0+3)^2 - 12 = -3$

$f(x) = [-3, \infty]$

(b) Find an expression for $f^{-1}(x)$. [2]

$y = (x+3)^2 - 12$

$y + 12 = (x+3)^2$

$\sqrt{y+12} = x+3 \Rightarrow x = \sqrt{y+12} - 3$

$f^{-1}(x) = \sqrt{\cancel{y}x+12} - 3$

(c) Solve the equation $gf(x) = 69$. [4]

$2\left([x+3)^2 - 12\right] - 5 = 69$

$2(x+3)^2 - 24 - 5 = 69$

$2x^2 + 12x + 18 - 29 = 69$

$2x^2 + 12x - 80 = 0$

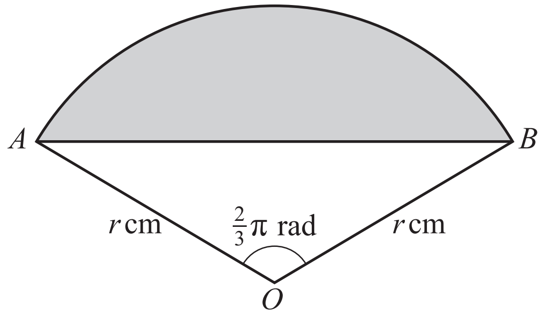
$x^2 + 6x - 40 = 0$

$x^2 + 10x - 4x - 40 = 0$

$x(x+10) - 4(x+10) = 0$

$x = 4, \quad x = -10$

7



The diagram shows a sector of a circle with centre O and radius r cm. The shaded region is bounded by the chord AB and the arc AB. The size of angle AOB is $\frac{2}{3}\pi$ radians.

(a) Show that the area of the shaded region is approximately $0.614r^2$ cm^2. [2]

Area of Region = Area of sector − area of Triangle OAB

Area of sector $= \frac{1}{2}r^2\theta = \frac{1}{2}r^2 \cdot \left(\frac{2}{3}\pi\right)$

$= \frac{r^2}{3}\pi$

area of Triangle $= \frac{1}{2} \times a \times b \sin\theta$

$= \frac{1}{2} \times r \times r \sin\frac{2\pi}{3}$

$= \frac{\sqrt{3}}{4} r^2$

Area of Region $= \frac{r^2}{3}\pi - \frac{\sqrt{3}}{4} r^2$

$r^2\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)$

$0.614 r^2$ proved

It is given that the radius of the circle is increasing at a rate of $0.4\,\text{cm}\,\text{s}^{-1}$.

(b) **(i)** Find the rate of increase of the area of the shaded region at the instant when $r = 20$. Give your answer correct to 2 significant figures. [3]

given $\frac{dr}{dt} = 0.4$ cm/s

$A = r^2\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)$

$\frac{dA}{dt} = 2r \cdot \frac{dr}{dt}\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)$

$= 2 \times 20 \times 0.4\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right) = 16 \times 0.614$

$= 9.8$

(ii) Find the rate of increase of the length of the arc AB. Give your answer correct to 2 significant figures. [3]

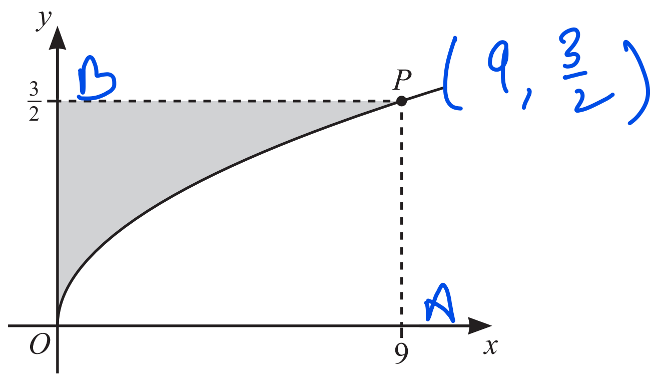
$l = r \times \theta$

$l = \frac{2\pi}{3} r$

$\frac{dl}{dt} = \frac{2\pi}{3} \times \frac{dr}{dt}$

$\frac{dl}{dt} = \frac{2\pi}{3} \times 0.4 = 0.83$

8



The diagram shows the curve with equation $y = \frac{1}{2}\sqrt{x}$ and the point P with coordinates $\left(9, \frac{3}{2}\right)$. The shaded region is bounded by the curve and the lines $x = 0$ and $y = \frac{3}{2}$.

(a) Find the area of the shaded region. [3]

Area of shaded region = area of Rectangle OAPB − area of curve OAP

area of Rectangle = $9 \times \frac{3}{2} = \frac{27}{2}$

area of curve = $\int_0^9 y \, dx$

$= \int_0^9 \frac{\sqrt{x}}{2} \, dx = \frac{1}{2}\left(\frac{x^{3/2}}{\frac{3}{2}}\right)_0^9$

$= \left(\frac{9^{3/2}}{3}\right)$

$= \cancel{30\times} \frac{27}{3} = \cancel{8} \; 9$

Area of shaded Region = $\frac{27}{2} - 9 = \frac{9}{2}$

(b) The shaded region is rotated through 360° about the ***y*-axis**.

Find the exact volume of the solid produced. [4]

Volume of revolution $= \pi \int x^2 \, dy$

$\sqrt{x} = 2y$

$x = 4y^2$

$= \pi \int_0^{3/2} (4y^2)^2 \, dy$

$= 16\pi \int_0^{3/2} y^4 \, dy$

$= 16\pi \left[\frac{y^5}{5} \right]_0^{3/2}$

$= \frac{16}{5}\pi \left[\left(\frac{3}{2}\right)^5 \right]$

$= \frac{16}{5} \times \pi \times \frac{243}{32_2} = \frac{243\pi}{10}$

9 An arithmetic progression has first term 2 and common difference d. The sum of the first n terms is denoted by S_n.

(a) It is given that $(S_2 - 1)$, S_4, S_9 are the first three terms of a second arithmetic progression.

Find the value of d. [4]

$a = 2$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_2 = \frac{2}{2}[2 \times 2 + (2-1)d]$

$= [4 + d$

$S_2 - 1 = 4 + d - 1 = d + 3 = -\frac{1}{5} + 3 = \frac{14}{5}$

$S_4 = \frac{4}{2}[2 \times 2 + (4-1)d]$

$= 2[4 + 3d] = 8 + 6d = 8 + 6 \times \frac{-1}{5} = \frac{34}{5}$

$S_9 = \frac{9}{2}[2 \times 2 + (9-1)d]$

$S_9 = 18 + 36d = 18 - \frac{36}{5} = \frac{90 - 36}{5} = \frac{54}{5}$

If a, b, c are in A.P then

$2b = a + c$ apply it

$2 \cdot S_4 = (S_2 - 1) + S_9$

$2(8 + 6d) = d + 3 + 18 + 36d$

$-25d = 5$

$d = \frac{-1}{5}$

(b) Hence find the difference between the values of the 15th terms of the two arithmetic progressions. [4]

$a = 2, \quad d = -\frac{1}{5}$

15th Term of AP1

$a_n = a + (n-1)d$

$a_{15} = 2 + (15-1) \times -\frac{1}{5}$

$= 2 - \frac{14}{5} = -\frac{4}{5}$

The AP2 has

$\frac{14}{5}, \frac{34}{5}, \frac{54}{5}$

$a = \frac{14}{5} \quad d = \frac{34}{5} - \frac{14}{5} = \frac{20}{5} = 4$

$a_{15} = a + (n-1)d$

$= \frac{14}{5} + (15-1) \times 4$

$\frac{14}{5} + 14 \times 4 = \frac{14 + 56 \times 5}{5}$

$a_{15} = \frac{280 + 14}{5} = \frac{294}{5} = 58.8$

$a_{15} = 58.8$

Diff $= 58.8 - \left(-\frac{4}{5}\right)$

10 A circle has equation $x^2+y^2+4y-21=0$ and a straight line has equation $2x+y-8=0$.
The line intersects the circle at two points.

(a) Find the coordinates of these two points of intersection. [4]

$x^2+(8-2x)^2+4(8-2x)-21=0$

$x^2+(64+4x^2-32x)+32-8x-21=0$

$5x^2-40x+75=0$

$x^2-8x+15=0$

$x^2-5x-3x+15=0$

$x(x-5)-3(x-5)=0$

$x=3,5$

when $x=3$

$2\times3+y-8=0 \Rightarrow y=2$

When $x=5$

$2\times5+y-8$

$y=-2$

$(3,2)$ $(5,-2)$

(b) The circle has centre C and the two points of intersection are denoted by A and B.

Find the area of the triangle ABC. [3]

Centre of the circle

$x^2+y^2+2gx+2fy+c=0$

$x^2+y^2+4y-21=0$

$2f=4$

$f=2$

$g=0$

Centre : $(0,-2)$

$A(3,2)$

$B(5,2)$

area of triangle ABC

$\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)$

$=\frac{1}{2}(0(2+2)+3(-2+2)+5(-2-2)$

$\frac{1}{2}[0+0-20]$ $=10$

11 A curve passes through the point $P(4, 3)$ and is such that

$$\frac{dy}{dx} = \frac{8}{x^2} - \frac{10}{(2x-3)^2}.$$

(a) Find the equation of the normal to the curve at P. Give your answer in the form $y = mx + c$. [3]

gradient at $P(4,3)$ is

$\left(\frac{dy}{dx}\right)(4,3) = \frac{8}{16} - \frac{10}{(2\times4-3)^2}$

$= \frac{1}{2} - \frac{10}{25} = \frac{1}{2} - \frac{2}{5}$

$= \frac{5-4}{10} = \frac{1}{10}$

gradient of normal $= -10$

$y - 3 = -10(x-4) \Rightarrow y = -10x + 40 + 3$

(b) Find the rate of change of the gradient of the curve when $x = 4$. [3]

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{8}{x^2} - \frac{10}{(2x-3)^2}\right]$

$= -2 \cdot \frac{8}{x^3} + \frac{10\times2}{(2x-3)^3} \times 2$

but $x = 4$

$= \frac{-16}{4^3} + \frac{40}{(2\times4-3)^3}$

$= \frac{-16}{64} + \frac{40}{125}$

(c) Given that the curve also passes through the point $(-1, q)$, find the value of q. [5]

Integrate $\frac{dy}{dx} = \frac{8}{x^2} - \frac{10}{(2x-3)^2}$

$$y = \int \frac{8}{x^2}\,dx - \int \frac{10}{(2x-3)^2}\,dx$$

$$y = 8\,\frac{x^{-2+1}}{-2+1} - 10\,\frac{(2x-3)^{-2+1}}{(-2+1)\times 2} + C$$

$$y = -8x^{-1} + 10(2x-3)^{-1} + C$$

put $(-1, q)$

$$q = -8(-1)^{-1} + 10(2\times -1 - 3)^{-1} + C$$

$$q = 8 + \frac{10}{-5} + C$$

$$q = 8 - 2 + C$$

$$C = q - 6$$

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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